A complete model of a tube amplifier stage

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1 Introduction

In this note we are going to build a mathematical model of a tube amplifier stage; as an example, a common cathode triode amplifier will be examined, but the exposed method can be applied to every other tube amplifier. In the first part of this note single tubes are studied; more in particular, an equivalent electrical model and a mathematical model are given. Then, in the second part, these models are used to determine the equation of a complete amplifier stage which contains a tube.

2 Triode Models

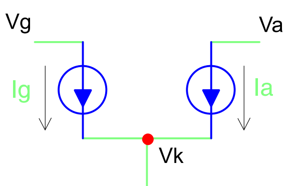
Before going into the analysis of the common cathode triode amplifier, we should describe the triode model that we are going to use. The model have been choosen according to the following observations:

1. We suppose the model is operating at audible frequencies, so it is not necessary to consider all the secondary effects, like the Miller capacity.

2. We are not interested into the heater model, because it is independent of the signals on the grid, anode and cathode.

3. The model should be generic enough to represent many different models of triodes.

The figure below depicts our choice for the model:



It is composed of two current sources, $i_g = i_g(v_{gk}, v_{ak})$ for the grid current, and $i_a = i_a(v_{gk}, v_{ak})$ for the anode current. Note that v_{gk}, voltage between grid and cathode, and v_{ak}, voltage between anode and cathode, are the two independent variables. We can model the behaviour of different troides by choosing the appropriate functions $i_g(v_{gk}, v_{ak})$ and $i_a(v_{gk}, v_{ak})$.

There are two approaches for choosing these functions:

1. By measurements on a real tube

2. With theoretic models

In the first method, we take a particular model of tube and perform many electrical measurements, inside all its operating range of the two parameters v_{gk} e v_{ak}; instead of measurements on the physical component, we could also obtain the same information using the curves on the data sheets of the component. Functions i_a and i_g can be obtained interpolating all the measured points. Interpolation with spline is suggested because of its properties of smoothness and regularity and its absence of oscillations.

The second method uses analytical models that approximate the real ones. We are going to introduce a couple of these models; for a complete list, see [1].

2.1 Leach model

According to the Leach model, the functions i_g e i_a are defined as follows:

$$i_a = \begin{cases} K(\mu v_{gk} + v_{ak})^{3/2} & , (\mu v_{gk} + v_{ak}) > 0 \\ 0 & , (\mu v_{gk} + v_{ak}) \leq 0 \end{cases}$$

$$i_g = \begin{cases} 0 & , v_{gk} < v_\gamma \\ \frac{v_{gk} - v_\gamma}{R_{gk}} & , v_{gk} > v_\gamma \end{cases}$$

By tweaking the parameters K, μ and v_γ different type of tubes can be modeled. These equations have a very simple formulation, but they are not so accurate and present discontinuity, which is a problem if they are used into a numerical method.

2.2 Rydel model

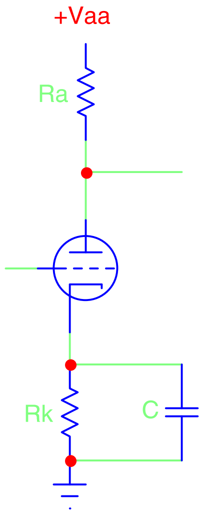
The Rydel model defines the functions i_g e i_a as follows:

$$i_a = K(1 + \frac{v_{gk}}{B})(v_{gk} + \frac{v_{ak} + v_c}{\mu})^{3/2}$$

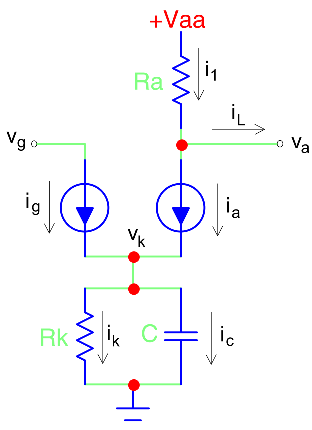
$$i_g = (\frac{A + v_{ak}}{B + v_{ak}})^4 v_{gk}^{3/2}$$

3 Building a model for the common cathode triode amplifier

In this section we would like to build the model for the following tube section:



which is a common cathode triode amplifier. Considering the triode model described in the previous section, the above schematic becomes:



where $v_g(t)$ is the voltage on the tube grid, $i_g(t)$ is the current flowing into the grid, $v_a(t)$ is the voltage on the anode and so the voltage at the output of the amplifier, and $i_L(t)$ is the current flowing out of the amplifier into the load. The equations we are looking for are simply a law between these variables. Note that the load at the output and the decoupling stage before the amplifier are not considered now; they will be added in the next section.

For simplicity, the time t dependency is omitted in the following equations:

$$\begin{cases} v_k = R_k i_k \\ v_k = \frac{1}{C}\int_0^t i_c(s)ds \\ i_g + i_a = i_k + i_c \\ i_1 = i_L + i_a(v_{ak}, v_{gk}) \\ V_{aa} - v_a = i_1 R_a \\ v_{ak} = v_a - v_k \\ v_{gk} = v_g - v_k \\ i_a = i_a(v_a - v_k, v_g - v_k) \\ i_g = i_g(v_a - v_k, v_g - v_k) \end{cases}$$

that after some algebra become:

$$\begin{cases} i_a = i_a(v_a - v_k, v_g - v_k) \\ i_g = i_g(v_a - v_k, v_g - v_k) \\ i_L = \frac{V_{aa} - v_a}{R_a} - i_a \\ i_k = \frac{1}{CR_k}\int_0^t (i_g + i_a - i_k)ds \\ v_k = R_k i_k \end{cases}$$

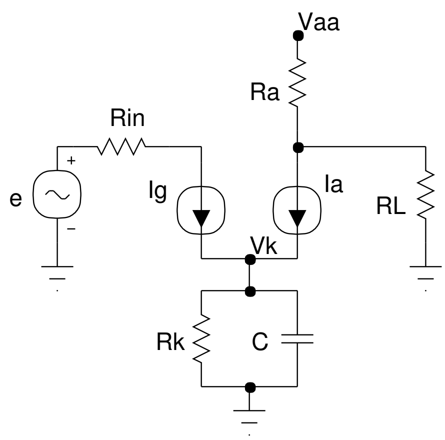
and finally:

$$\begin{cases} i_k = \frac{1}{CR_k}\int_0^t ((i_a + i_g)(v_a - R_k i_k, v_g - R_k i_k) - i_k)ds \\ i_g = i_g(v_a - R_k i_k, v_g - R_k i_k) \\ i_L = \frac{V_{aa} - v_a}{R_a} - i_a(v_a - R_k i_k, v_g - R_k i_k) \end{cases}$$

i_k is a status variable and it is necessary to hold the "memory effect" of the capacitor C.
These are the relations between the four variables $v_g(t)$, $i_g(t)$, $v_a(t)$ and $i_L(t)$. But these are not solvable ODEs, because they are modeling an opened circuit.

4 The basic circuit

To transform these relations into the equations of a circuit, we should add a load at the output and a wave source at the input, like in the circuit below:



The voltage generator $e(t)$ represent the signal at the input at the tube stage, so we call its value v_{in}. The output of the stage is v_a, the voltage on the load resistor R_L. The equations for this circuit can be easily written starting from the equations in the previous section and adding into the system the new relations between the input current and input voltage, and between the output current and the output voltage.

$$\begin{cases} v_{in} = R_{in} i_g + v_g \\ i_k = \frac{1}{CR_k} \int_0^t ((i_a + i_g)(v_a - R_k i_k, v_g - R_k i_k) - i_k) ds \\ i_g = i_g(v_a - R_k i_k, v_g - R_k i_k) \\ i_L = \frac{V_{aa} - v_a}{R_a} - i_a(v_a - R_k i_k, v_g - R_k i_k) \\ v_a = i_L R_L \end{cases}$$

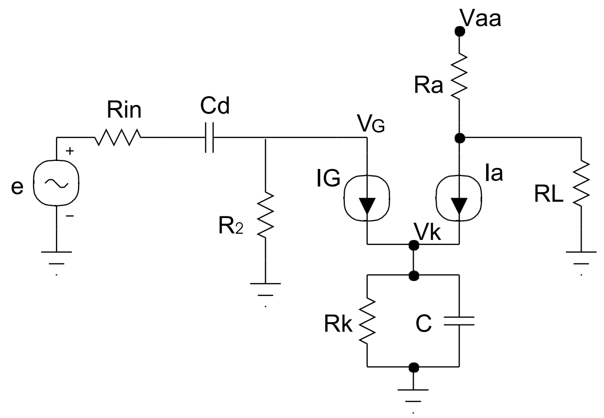
that, substituting v_g from the first equation and v_a from the last one, becomes:

$$\begin{cases} i_k = \frac{1}{CR_k} \int_0^t ((i_a + i_g)(i_L R_L - R_k i_k, v_{in} - R_{in} i_g - R_k i_k) - i_k) ds \\ i_g = i_g(i_L R_L - R_k i_k, v_{in} - R_{in} i_g - R_k i_k) \\ i_L = \frac{V_{aa} - i_L R_L}{R_a} - i_a(i_L R_L - R_k i_k, v_{in} - R_{in} i_g - R_k i_k) \end{cases}$$

Now, this is a system of three equations in the three unknown i_g, i_L, i_K. Knowing i_L we can immediately obtain the output voltage from $v_a = i_L R_L$.

5 The complete circuit

The circuit in the previous section is not what we usually find in real case. Instead we often see something like this:



The condenser C_d has a decoupling function between the current stage and the previous one; it is configured as a high pass filter. It is very important to note that this is true only if we suppose that the grid of the triode does not sink current, but actually this is not true. When the tube is saturating on its positive halfwave, there is a current flowing into the grid so the capacitor is going to charge; all this process causes a shift in the bias point of the grid and consequently the way the tube is saturating. From a musical point of view, this means that this tube stage produces more harmonics during the transients. This also means that tubes cannot be treated as simple waveshapers, like many people may think!

The equations for this new circuit are:

$$\begin{cases} i_1 = i_g + \frac{v_g}{R_2} \\ v_{in} = i_1 R_{in} + v_{C_d} + v_g \\ v_{C_d} = \frac{1}{C_d} \int_0^t i_1 ds \\ i_k = \frac{1}{CR_k} \int_0^t ((i_a + i_g)(v_a - R_k i_k, v_g - R_k i_k) - i_k) ds \\ i_g = i_g(v_a - R_k i_k, v_g - R_k i_k) \\ i_L = \frac{V_{aa} - v_a}{R_a} - i_a(v_a - R_k i_k, v_g - R_k i_k) \\ v_a = i_L R_L \end{cases}$$

that after some algebra become:

$$\begin{cases} v_{in} = (i_g + \frac{v_g}{R_2}) R_{in} + \frac{1}{C_d} \int_0^t (i_g + \frac{v_g}{R_2}) ds + v_g \\ i_k = \frac{1}{CR_k} \int_0^t ((i_a + i_g)(v_a - R_k i_k, v_g - R_k i_k) - i_k) ds \\ i_g = i_g(v_a - R_k i_k, v_g - R_k i_k) \\ i_L = \frac{V_{aa} - v_a}{R_a} - i_a(v_a - R_k i_k, v_g - R_k i_k) \\ v_a = i_L R_L \end{cases}$$

Watching at this equations, is there someone still considering a tube stage like a waveshaper?

6 Acknowledgment

I would like to thank Gianpaolo Borin who inspired this work.

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